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ENGINEERING CALCULATION OF THE CHARACTERISTICS OF A FILTRATION-COMBUSTION WAVE BASED ON A ONE-DIMENSIONAL TWO-TEMPERATURE MODEL

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The authors present a method for determining the characteristics of a filtration-combustion wave that uses physically substantiated simplifications of a one-dimensional two-temperature model of filtration flow with Arrhenius kinetics. Explicit, physically transparent dependences for the ignition temperature, wave velocity, and maximum skeleton temperature are obtained. A brief review of and a comparison with other one-dimensional methods and with a numerical calculation are given. This method is shown to be superior to more cumbersome analytical two-temperature methods in accuracy.

The process of combustion in a porous medium, or filtration combustion (FC), attracts great interest of researchers in connection with numerous technical applications of this process such as recycling (oxidation) of noxious gases, combustion of lean fuel mixtures in a superadiabatic-combustion regime, thermal-wave treatment of catalysts, and others [1, 2].

When real devices with FC are developed, understanding of the occurring processes and accessible apparatus for calculating the basic characteristics of FC waves are required foremost. The majority of works on FC of gases, on the other hand, contain cumbersome formulas of little use for engineering calculations. In the present work, an attempt is made to provide a more general picture of the methods of analysis of one-dimensional stationary FC of gases in an inert skeleton, and a simple explicit formula to calculate the ignition temperature and the maximum skeleton temperature is derived. This approach is based on physically substantiated simplifications of a one-dimensional two-temperature model of filtration combustion with Arrhenius kinetics. Problems of unsteadiness and wave stability and other more complicated issues are not considered in this paper.

In [3, 11], regimes of slow filtration combustion and rapid (in fact, break of the flame through the pore space) filtration combustion are distinguished. By FC we will mean a slow-combustion regime, where intense interphase heat transfer governs the basic regularities of combustion-wave propagation.

The issues of analytical and numerical descriptions of filtration-combustion processes have been the objective of numerous works, references to which can be found in [2-4] and other works. In analysis and mathematical modeling, in principle, there can be two approaches to the statement and solution of the problem:

1) *volume-averaged modeling*, in which use is made of the effective (averaged over the volume) thermophysical characteristics of the porous skeleton and the gas, and the interphase interaction is described by the heat transfer coefficient α;

2) *direct point modeling*, in which the energy and momentum equations (for the gas) are considered and the boundary conditions are established at the interface.

Direct point modeling was investigated in the works of Kaviani et al. [4, 5]. As a result it was shown that this approach has no advantages over the volume-averaged approach when FC macroscopic parameters (structure, thickness, velocity, characteristic temperatures of the FC wave) are determined, although it predicts intrapore variations in the gas temperature up to 40% and in the combustion rate up to 20% [5]. Since results on intrapore thermal processes, in practice, are not checkable experimentally, it is unreasonable to use direct point modeling to

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investigate FC. An important drawback of it is computational tediousness and the impossibility of considering the FC problem successively and analytically. Therefore, in the majority of cases, the volume-averaged approach is used.

Methods for Analyzing One-Dimensional Problems of FC of Gases. In considering FC of gases, we usually solve a system of balance equations for the porous-body temperature, the gas temperature, and the concentration of the deficient reagent:

$$(c\rho)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\lambda\frac{\partial T}{\partial x} - \alpha(T - T_g) - \beta(T - T_0), \tag{1}$$

$$(c\rho)_g\frac{\partial T_g}{\partial t} + (c\rho)_g u_g\frac{\partial T_g}{\partial x} = \frac{\partial}{\partial x}\lambda_g\frac{\partial T_g}{\partial x} + \alpha(T - T_g) + QW(y, T_g), \tag{2}$$

$$\frac{\partial y}{\partial t} + u_g\frac{\partial y}{\partial x} = \frac{\partial}{\partial x}D\frac{\partial y}{\partial x} - W(y, T_g). \tag{3}$$

Here the basic notation is standard and is given at the end of the paper. The subscript g denotes the gas component; the solid-phase parameters are without subscripts. The value of the porosity m is involved implicitly via the heat capacities, thermal conductivities, and heat-transfer coefficient.

The possibilities of analytical solution of such systems are limited, although important qualitative and quantitative relations can be obtained on the basis of certain model assumptions. Thus, considering the energy balance of the FC wave, we can show that the maximum equilibrium temperature of the steadily propagating wave in the absence of heat losses is related to this wave's velocity:

$$\Delta T_{max} = \frac{\Delta T_{ad}}{1 - \dfrac{u_w}{u_t}}, \tag{4}$$

where ΔT_{max} is the maximum (equilibrium with the gas) temperature in the combustion wave; ΔT_{ad} is the adiabatic temperature of fuel-mixture combustion; u_t is the wave velocity in the absence of a reaction of heat release, determined as $u_t \equiv u_g(c\rho)_g/c\rho$; u_w is the actual velocity of the FC front. From (4), it can be seen that ΔT_{max} can substantially exceed the adiabatic combustion temperature in the case of co-motion of the combustion wave and is always lower than ΔT_{ad} in the case of countermotion of the wave. However without detailing the mechanisms of heat transfer and heat release it is impossible to determine important FC parameters such as u_w, ΔT_{max}, and others.

In analyzing FC, wide acceptance has been received by *a one-temperature approximation* in which the gas T_g and solid-phase T temperatures are assumed to be equal due to the high intensity of the interphase heat transfer, $\alpha \to \infty$. The two energy-balance equations (1) and (2) are reduced to one:

$$(c\rho + (c\rho)_g)\frac{\partial T}{\partial t} + u_g(c\rho)_g\frac{\partial T}{\partial x} = \frac{\partial}{\partial x}(\lambda + \lambda_g)\frac{\partial T}{\partial x} - \beta(T - T_0) + QW(y, T). \tag{5}$$

If the FC wave propagates with a constant velocity u_w, then in a coordinate system that moves with the wave, Eq. (5) takes the form

$$-u_w(c\rho + (c\rho)_g)\frac{\partial T}{\partial x} + u_g(c\rho)_g\frac{\partial T}{\partial x} = \frac{\partial}{\partial x}(\lambda + \lambda_g)\frac{\partial T}{\partial x} - \beta(T - T_0) + QW(y, T), \tag{6}$$

or, by introducing the thermal velocity of the front $u_t = u_g(c\rho)_g/c\rho$ and the dimensionless velocity of the front of the combustion wave $u \equiv u_w/u_t$, we have

$$u_g (c\rho)_g \left(1 - u - \frac{u_w}{u_g}\right) \frac{\partial T}{\partial x} = \frac{\partial}{\partial x} (\lambda + \lambda_g) \frac{\partial T}{\partial x} - \beta (T - T_0) + QW(y, T) . \tag{7}$$

Since $u_w/u_g \sim 10^{-3}$ and $\lambda_g << \lambda$, the corresponding terms of Eq. (7) are not considered in its integration.

The *instantaneous-reaction* approximation assumes an infinitely fast reaction of combustion when the ignition temperature T_i is attained by the gas:

$$W(y, T) = \delta (T_i) .$$

We note that, in the one-temperature approximation, provided that the reaction is instantaneous, the ignition temperature T_i for the gas and the solid phase is the same and it is the maximum temperature in the FC wave. In this approximation, a relation that is convenient for practical purposes and relates the dimensionless velocity of the FC front to the ignition temperature and the parameters of heat losses was obtained [8]:

$$u \equiv \frac{u_w}{u_t} = 1 - \sqrt{\left(\left(\frac{\Delta T_{ad}}{\Delta T_{max}}\right)^2 - \frac{4\beta a}{\alpha}\right)} , \tag{8}$$

where $a = \alpha\lambda/u_g^2 (c\rho)_g^2$. The values of temperature-dependent quantities are taken at the maximum temperature in the combustion wave.

In the indicated approximation, disregarding the volume heat losses, $\beta = 0$, we can easily obtain stationary profiles for plane, spherical, and cylindrical geometries of the FC front. Thus, for a plane wave and a constant thermal conductivity λ = const, integrating (7) yields

$$T = T_0 + \Delta T_i \exp\left(\frac{u_g (c\rho)_g (1 - u)}{\lambda} (x - x_i)\right) , \quad x < x_i , \tag{9}$$

where x_i is the coordinate of the combustion front. In the case of a coefficient λ that corresponds to radiation diffusion $\lambda = \lambda_0 T^3$, integrating (7) yields

$$x - x_i = \frac{\lambda_0 T_0^3}{u_g (c\rho)_g (1 - u)} \left(\frac{T - T_i^3}{3T_0^3} + \frac{T - T_i^2}{2T_0^2} + \frac{T - T_i}{T_0} + \ln \frac{T - T_0}{T_i - T_0}\right) , \quad x < x_i . \tag{10}$$

For cylindrical and spherical Laplacians, integration over the radial coordinate from 0 to r_i leads to the solutions

$$T = T_0 + (T_i - T_0) (r/r_i)^b , \quad r < r_i \tag{11}$$

and

$$T = T_0 + (T_i - T_0) \exp(b'/r_i - b'/r) , \quad r < r_i , \tag{12}$$

respectively. Here $b = c_g G_0/(2\pi L\lambda)$, $G_0 = 2\pi r L \rho_g u_g$ is the mass flow rate, L is the cylinder length, $b' = c_g G_0'/(4\pi\lambda)$, $G_0' = 4\pi r^2 \rho_g u_g$ is the mass flow rate in the case of spherical geometry; λ is constant everywhere.

Although the instantaneous-reaction approximation enables us to integrate (7) analytically, it is important that the problem becomes subdefined and admits solutions with different maximum temperatures in the wave. To avoid arbitrariness, the maximum (or a different characteristic) temperature in the wave must be determined from additional physical models. In the two-temperature model, the porous-medium temperature that corresponds to fuel ignition $T_{s,i}$ (chemical-reaction "initiation") is most often chosen as the characteristic temperature.

In [13], a solution to the problem in the one-temperature approximation is obtained using the Frank-Kamenetskii approximation for the Arrhenius reaction rate. Here, the methods of so-called counterextrapolation and joined asymptotic expansions are used.

With allowance for the *chemical kinetics of the reaction in the form of the Arrhenius law*, it is impossible to obtain an exact analytical solution in finite functions even in the one-temperature approximation. However in [6], for Arrhenius kinetics, the authors were able to find an analytical solution that approximates the exact solution accurate within 1–2% in a wide parameter range.

In a number of works, analytical solutions of the *two-temperature* problem (1)-(3) are given. In [3], a two-temperature solution in the instantaneous-reaction approximation was analyzed. In the same work, a similar problem with allowance for external heat losses described by Newtonian heat transfer was solved. To determine the ignition temperature $T_{s,i}$, the authors of [3] draw on a formal analogy with the problem of a thermal explosion where a dimensionless space coordinate is interpreted as dimensionless time. By using the condition of critical heat losses from the combustion region, a nonlinear equation is obtained that determines the wave temperature and velocity. In [3], there are unclear substitutions and variables, which makes it difficult to use its results for practical calculations.

In [7], a two-temperature solution of the FC problem in the instantaneous-reaction approximation is given; however, the issue of initiation of the chemical reaction is not considered, owing to which macroscopic characteristics of the combustion wave – velocity, maximum temperature – are not determined.

In [8], the two-temperature problem is successively analytically solved in the instantaneous-reaction approximation with allowance for external heat losses. To determine the temperature of reaction initiation, the authors draw on the notion of a thermal explosion of an element of the gas volume that has the characteristic dimension of the interpore space with allowance for reagent burnup. They obtained the expression

$$\frac{\exp\left(E/RT_{s,i}-1\right)}{\left(E/RT_{s,i}\right)^{7/3}}=\left(\frac{\Delta T_{ad}^{2}R^{5}}{2\pi^{2}E^{5}}\right)^{1/3}\frac{d_0 m T_0 K}{u_{g,0}}, \tag{13}$$

where m is the porosity; $u_{g,0}$ is the filtration rate at the initial temperature; d_0 is the diameter of a porous-filling particle; $T_{s,i}$ is the ignition temperature. Among the drawbacks of evaluations of the ignition temperature $T_{s,i}$ of the type (13) [3, 8] are:

1) the absence of parameters of interphase heat transfer and thermophysical characteristics of the solid phase in the equations;

2) in deriving the formulas, the temperatures of the gas and solid phases are taken to be the same, although in problems of ignition, they are physically unequal [9];

3) insufficient substantiation of using the approaches of thermal-explosion theory to determine the ignition temperature $T_{s,i}$. Indeed, the use of thermal-explosion theory is reasonable if the time τ_{ad} and space scales of the Zel'dovich–Frank-Kamenetskii theory are of the order of or smaller than the characteristic scales of the FC problem, determined by the pore dimensions d_0 and the gas-flow velocity (actually, these conditions are not satisfied for high pumping rates and low calorific power of the gases at low porosity, high heat losses, etc.) and when the condition of smallness of the parameters RT_{ad}/E and the burnup parameter is satisfied;

The formulas obtained as a result of successive two-temperature solution of the problem are cumbersome, require numerical solution of nonlinear equations or systems of equations for determining the ignition temperature and other parameters, and owing to this are of little use for engineering calculations of the characteristics of an FC wave.

Simplified Two-Temperature Model of FC. We give a simplified physical approach to the solution of the FC problem based on the two-temperature approximation. We formulate the problem of gas ignition in the following manner: we assume that the chemical heat release is activated when the flux of interphase convective heat transfer $q_{s,g}$ is equal in magnitude to the chemical heat release flux:

$$q_{s,g}=\alpha\Delta T_{s,g}=Qy_0 z\exp\left(-E/R\left(T_0+\Delta T_{s,i}-\Delta T_{s,g}\right)\right), \tag{14}$$

where $\Delta T_{s,g}$ is the interphase temperature difference at the firing point; $\Delta T_{s,i}$ is the skeleton temperature that corresponds to ignition (see the diagram of Fig. 1).

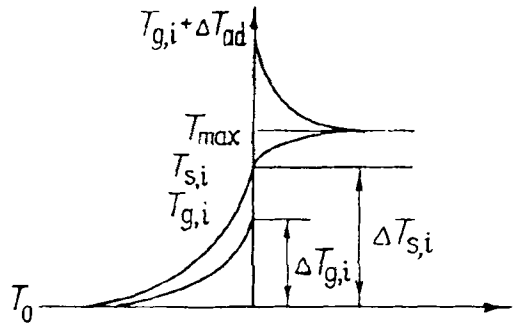

Fig. 1. Characteristic temperatures of a two-temperature FC model.

According to the two-temperature analytical solution to the problem [3, 7, 8], the temperature profiles for the solid and gas phases are similar, i.e., are prescribed by the same exponents in the region where the chemical heat release can be disregarded. Taking advantage of this property, we assume that the exponent corresponds to the one-temperature solution (9) and express the temperature gradient of the gas at the firing point:

$$\frac{\partial T_g}{\partial x} = (\Delta T_{s,i} - \Delta T_{s,g}) \frac{u_g (c\rho)_g (1-u)}{\lambda}. \tag{15}$$

Disregarding the thermal conductivity and the heat capacity of the gas as compared to the corresponding parameters of the solid phase as well as the chemical heat release in the inert zone, from (2) we obtain

$$\frac{\partial T_g}{\partial x} = \frac{\alpha \Delta T_{s,g}}{u_g (c\rho)_g}. \tag{16}$$

By setting (15) equal to (16), we find

$$\Delta T_{s,g} = \frac{\Delta T_{s,i}}{1 + \dfrac{\alpha\lambda}{u_g^2 (c\rho)_g^2 (1-u)}} \tag{17}$$

or, by using (4),

$$\Delta T_{s,g} = \frac{\Delta T_{ad}}{\dfrac{\Delta T_{ad}}{\Delta T_{s,i}} + \dfrac{\alpha\lambda}{u_g^2 (c\rho)_g^2}}. \tag{18}$$

From (17) and (18), it follows that $\Delta T_{s,g} < \Delta T_{s,i}$ for any rate of gas filtration u_g; however, the value of the interphase temperature difference $\Delta T_{s,g}$ is not bounded, since $\Delta T_{s,i}$ and $\Delta T_{s,g}$ increase monotonically with the filtration rate. In fact, $\Delta T_{s,g}$ cannot exceed ΔT_{ad} for any u_g (since, in this case, the heat source for the solid phase disappears), we correct (18) by making the assumption that $\Delta T_{ad}/\Delta T_{s,i} = 1$. To eliminate this contradiction, we correct (18), making the assumption $\Delta T_{ad}/\Delta T_{s,i} = 1$. The substantiation of this assumption is that: 1) for low filtration rates u_g, it does not affect the accuracy of (18) since $(\Delta T_{ad}/\Delta T_{s,i}) << (\alpha\lambda/u_g^2(c\rho)_g^2)$; 2) for medium and high filtration rates, it corrects (18) adequately in the sense that it eliminates the aforementioned physical contradiction. Further analysis and testing showed that this assumption is critical for improving accuracy of the model for high filtration rates. Therefore, we have

$$\Delta T_{s,g} = \frac{\Delta T_{ad}}{1 + \dfrac{\alpha\lambda}{u_g^2 (c\rho)_g^2}}. \tag{19}$$

Substituting (19) into (14), we obtain

$$T_{s,i} \equiv \Delta T_{s,i} + T_0 = \frac{\Delta T_{ad}}{1 + \dfrac{\alpha\lambda}{u_g^2 (c\rho)_g^2}} + \frac{E}{R}\left[\ln\left(\frac{Qz}{\alpha}\frac{\left(1 + \dfrac{\alpha\lambda}{u_g^2 (c\rho)_g^2}\right)}{\Delta T_{ad}}\right)\right]^{-1}. \tag{20}$$

Within the framework of this approach the first term of Eq. (20) is interpreted as the temperature difference between the solid and gas phases at the firing point, and the second term is the absolute temperature of the gas phase that corresponds to ignition of the mixture $T_{g,i}$, i.e., $T_{s,i} \equiv \Delta T_{s,i} + T_0 = \Delta T_{g,s} + T_{g,i}$.

The most important FC parameter – the wave velocity – can be calculated from Eq. (4) or, under conditions of substantial volumetric heat losses, from (8). In (4) and (8) we can use the ignition temperature $\Delta T_{s,i}$ as the maximum temperature, although it is obvious that the maximum temperature ΔT_{max} exceeds the ignition temperature by some quantity $\Delta T_{s,i-max}$. We evaluate it at a stationary point since near-stationary solutions are of greatest interest for engineering calculations of FC of gases. The solution to the problem within the framework of the two-temperature approximation and instantaneous reaction in view of estimate (19) for the magnitude of the temperature difference yields

$$\Delta T_{s,i-max} = \frac{\Delta T_{ad}}{\left(1 + \dfrac{1}{a}\right)\left(\dfrac{\sqrt{a}}{2} + \sqrt{\left(\dfrac{a}{4} + 1\right)}\right)}, \tag{21}$$

where a is the dimensionless combination in (8). From (21) it follows that $\Delta T_{s,i-max}$ is a dome-shaped function of a that vanishes as $a \to 0$ and $a \to \infty$, has a maximum for $a \sim 1$, and does not exceed $\sim \Delta T_{ad}/5$. We note that the latter property is universal in character since does not depend on the kind of fuel or the FC parameters; therefore, within the framework of the accuracy adopted in this paper, the estimate

$$\Delta T_{s,i-max} \cong 0.1 \Delta T_{ad} \tag{22}$$

will be reasonable. The quantity $\Delta T_{s,i-max}$ can be calculated by solving the nonlinear system of equations (20) and (21) if it is assumed in (20) that, in the stationary situation, $\Delta T_{s,i} + \Delta T_{s,i-max} = \Delta T_{ad}$. However the resultant formulas are cumbersome, and the gain in the accuracy of determining T_{max} is slight.

In the case of cylindrical or spherical geometry of the reactor with fuel filtration from the center to the periphery, it is impossible to go over to uniformly moving coordinates; however, the stationary situation can be considered similarly to how it was considered for a plane wave.

We use the one-temperature stationary solutions for the wave's inert zone (11) and (12) as the approximation for the skeleton temperature and assume that the gas temperature in the inert zone is described by a similar function: $T = T_0 + \Delta T_{s,i}(r/r_i)^b$ for the skeleton, $T_g = T_0 + (\Delta T_{s,i} - \Delta T_{s,g})(r/r_i)^b$ for the gas for a cylinder, and $T = T_0 + \Delta T_{s,i} \exp(b'/r_i - b'/r)$ for the skeleton, $T = T_0 + (\Delta T_{s,i} - T_{s,g}) \exp(b'/r_i - b'/r)$ for the gas for a sphere, where the notation corresponds to (11) and (12).

Using condition (14) as the ignition criterion and repeating computations (15)-(17), we arrive at a formula for the ignition temperature that is similar to (18), with the only difference that $\Delta T_{s,i}$ is substituted for ΔT_{ad}:

$$T_{s,i} \equiv \Delta T_{s,i} + T_0 = \frac{\Delta T_{s,i}}{1 + \dfrac{\alpha\lambda}{u_g^2 (c\rho)_g^2}} + \frac{E}{R}\left[\ln\left(\frac{Qz}{\alpha}\frac{\left(1 + \dfrac{\alpha\lambda}{u_g^2 (c\rho)_g^2}\right)}{\Delta T_{s,i}}\right)\right]^{-1}. \tag{23}$$

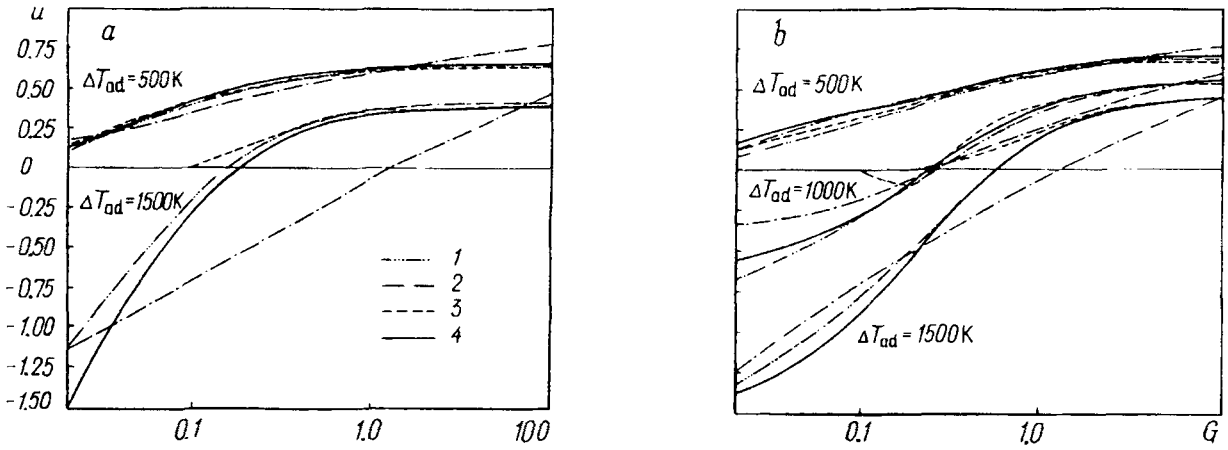

Fig. 2. Dimensionless velocity of an FC wave vs. the mass flow rate of a fuel G: 1) calculation according to the model (4), (20), and (22); 2) according to a one-temperature model; 3) according to a two-temperature model of a temperature "jump"; 4) direct integration of (1)-(3) [10]. The diameter of the granules of the filling is d_0 = 5.6 mm (a) and 1 mm (b).

We note that $\Delta T_{s,i}$ can be expressed in explicit form if $\Delta T_{s,i} = \Delta T_{ad}$ is assumed in the logarithmic function.

The nontrivial fact that the form of the ignition-temperature function for a sphere, a cylinder, and a plane is the same, in spite of the substantial difference in the temperature profiles means physically that the interphase temperature difference and the ignition temperature itself are governed solely by local characteristics of the system at the firing point: the local temperature, the local filtration rate, and the local heat capacity, thermal conductivity, and heat-transfer coefficient *and do not depend* on the time and temperature prehistory of the gas particle. This result is a consequence of the Arrhenius gross kinetics adopted by us and disregard of the reagent burnup at the firing point.

Comparison of FC Models and Recommendations for Their Application. To check the simplified model and assess the field of its application, we calculated the FC wave velocity for two sizes of the granules of the porous filling: 1 mm and 5.6 mm and a wide range of flow rates of the gas $G = 0.01-10$ kg/(sec·m^2) and its heat content $\Delta T_{ad} = 500-1500$ K. Velocities calculated according to (4), (20), and (22) were compared to the corresponding velocities obtained according to the one-temperature model and the two-temperature model of a temperature "jump" and as a result of solving the system (1)-(3). A computational algorithm for the latter three models is described in [10]. In all the calculations, the heat capacities and thermal conductivities of the gas and the skeleton were assumed constant, and lateral heat losses were disregarded. The results obtained are presented in Fig. 2.

The calculations performed enable us to make the following conclusions:

1. The one-temperature model can be used for engineering calculations of the FC wave for small sizes of the skeleton pores (grains) $d_0 \leq 1$ mm, a low heat content of the fuel $\Delta T_{ad} \leq 1000$, and low and medium filtration rates (flow rates) $G \leq 1$ kg/(sec·m^2). Otherwise, the miscalculation of the flow rate that corresponds to the cessation (zero velocity) of the wave G_{st} can attain an order of magnitude (Fig. 2a). In the region of high flow rates $G > 10G_{st}$, the one-temperature model predicts a linear increase in the wave temperature and rapid approach of the combustion-wave velocity to the thermal-wave velocity u_t, while, actually, the temperature increase slows down sharply and becomes logarithmic, and, accordingly, the increase in the wave velocity slows down, not exceeding 0.4–0.7 of u_t for any practicable flow rates of the gas. For extremely low heat content of the fuel $\Delta T_{ad} \lesssim 100$ K, good adequacy of the one-temperature model can be expected.

2. The two-temperature model of a temperature "jump" (the instantaneous-reaction approximation) describes well FC for medium and high flow rates, i.e., regimes of co-combustion. However, it does not describe regimes of countercombustion, and, for flow rates that are close to the cessation of the wave, it can yield a nonmonotonic dependence for u (and T_{max}) on the flow rate (Fig. 2b). The miscalculation of the flow rate G_{st} can attain 100%.

3. The model proposed in this work has an error of no more than 15–25% (relative to the numerical calculation) for determining the characteristic temperatures and FC-wave velocity over a wide range of flow rates, fuel heat contents, and characteristic pore dimensions. It can be used for qualitative and quantitative analyses of FC problems, including finding the FC-wave velocity and the characteristic temperatures of the combustion wave.

CONCLUSION

The problem of FC of gases is extremely complicated, since the multidimensional nature of the FC system, the complex mechanism of the chemical reaction, and the unsteadiness and inhomogeneity of the physical properties of the medium can have an effect in practice, not to mention the fact that the heat capacity and thermal conductivity of particular porous systems are usually known accurate to within 10–20%. Therefore, cumbersome and detailed models (in essence, remaining approximate) are justified only for specific statements of the problems and academic investigations of filtration combustion. On the other hand, crude one-dimensional models frequently not only describe the picture of FC-wave propagation qualitatively but also provide sufficient accuracy of determining temperature and dynamic characteristics of the wave. This is due to a certain monotonicity and smoothness of the basic functions that determine FC. Therefore simple models that yield explicit parametric dependences are of significant usefulness for practical engineering calculations. Here, the research engineer must correctly assess the field of application of the model.

As has been indicated above, the form of functions (20) and (23), to a certain degree, is due to our adoption of Arrhenius gross kinetics and disregard of the reagent burnup at the firing point. Therefore when low-calorie fuels are used near their explosive limit the issue of the accuracy of this approach (as of any other models that use the same assumptions) calls for additional investigation. It should also be borne in mind that when high-calorie fuels and skeletons with a large pore size are used the physical pattern of combustion can change and there will be a changeover to a regime of high velocities.

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NOTATION

T, skeleton temperature; T_0, ambient temperature; ΔT, change in the temperature relative to T_0; T_{max}, maximum skeleton temperature; T_{ad}, adiabatic temperature of fuel combustion; u_t, thermal-wave velocity; u_w, FC-front velocity; u_g, mean velocity of gas filtration; G_0, mass flow rate of the gas; y, concentration of the deficient reagent; $W(y, T)$, chemical-reaction rate; E, activation energy; R, universal gas constant; z, pre-exponent of the Arrhenius function; Q, thermal effect of the reaction; c, heat capacity at constant pressure; ρ, bulk density; λ, thermal conductivity; α, volumetric heat-transfer coefficient; β, coefficient of volumetric heat losses; m, skeleton porosity; d_0, diameter of a porous-skeleton grain; G, specific gas flow rate; r, radial coordinate; a, dimensionless combination in (8) and (21); b and b', parameters in (11) and (12); $\delta(x)$, delta function. Subscripts: g, gas phase; s, solid phase; i, ignition; 0, initial moment; st, steady state.

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